Propagation of EM Wave in the Guided Media

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Introduction

- A transmission line can be used to guide EM energy from one point (generator) to another (load).
- A waveguide is another means of achieving the same goal. It is a device to transmit EM waves (or energy) from one place to another.
- However, a waveguide differs from a transmission line in some respects, although we may regard the latter as a special case of the former.

<table>
<thead>
<tr>
<th>Transmission line</th>
<th>Wave guide</th>
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</thead>
<tbody>
<tr>
<td>• a transmission line can support only a transverse electromagnetic (TEM) wave</td>
<td>• a waveguide can support many possible field configurations</td>
</tr>
<tr>
<td>• at microwave frequencies (roughly 3-300 GHz) transmission lines become inefficient due to skin effect and dielectric losses</td>
<td>• waveguides are used at that range of frequencies to obtain larger bandwidth and lower signal attenuation.</td>
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<tr>
<td>• a transmission line may operate from dc ($f = 0$) to a very high frequency</td>
<td>• a waveguide can operate only above a certain frequency called the cutoff frequency and therefore acts as a high-pass filter.</td>
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</table>

Thus, waveguides cannot transmit dc, and they become excessively large at frequencies below microwave frequencies.
Types of Waveguides (WG)

Although a waveguide may assume any arbitrary but **uniform cross section**, common waveguides are either rectangular or circular.

Fig. Types of waveguides
Importance and Applications of WG

**Importance:**
- They are guided so there are no chance of leakage of energy.
- They can handle high power.
- They provide wide bandwidth.
- They are strong in structure and durable as well.
- Least amount of loss in high frequency application such as mm wave application.
- Inherently behaves as a high pass filter and many more.

**Applications:**
- Simply as a transmission line.
- Various passive components such as filter, coupler, divider, horn antennas, tee junction etc can be made out of waveguide.
- In high power application.
- In very high frequency application to avoid losses.
TEM, TE & TM Waves

**TEM Waves:**
- E-and H-Fields are perpendicular to each other.
- Both fields are transverse to the direction of propagation.
- No filed component are in the direction of propagation.

**TE Waves:**
- Transverse electric waves (TE-waves). \( E = (E_x; E_y; 0) \) and \( H = (H_x; H_y; H_z) \).
- No electric field components are in the direction of propagation.

**TM Waves:**
- Transverse magnetic waves (TM-waves). \( E = (E_x; E_y; E_z) \) and \( H = (H_x; H_y; 0) \).
- No magnetic field components are in the direction of propagation.

Fig. 2. Rectangular WG Wave propagating to z direction
Rectangular Waveguide

- Metallic pipe with rectangular cross-section.
- Two pairs of conducting plates.
- The guide is finite in $x$- and $y$-directions.
- TE and TM waves will propagate; TEM waves will not propagate.
Rectangular WG: Waveguide modes

- Electromagnetic waves can travel along waveguides using a number of different modes. The different waveguide modes have different properties.

**The Modes are:**

- **TE Mode:** No electric field components are in the direction of propagation
- **TM Mode:** No magnetic field components are in the direction of propagation
- **TEM Mode:** No field components are in the direction of propagation. The Transverse electromagnetic wave cannot be propagated within a waveguide.

- WG operating in TE and TM modes acts like a high pass filter, where the operating frequency is higher than cutoff frequency.

- For each waveguide mode there is a definite lower frequency limit. This is known as the cut-off frequency. Below this frequency no signals can propagate along the waveguide. The cutoff frequency depends on the WG dimension and propagation mode.
Rectangular WG: Why TEM Mode Not Exist in WG?

- Wave travel along a standard, two-conductor transmission line is of the TEM (Transverse Electric and Magnetic) mode, where both fields are oriented perpendicular to the direction of travel. TEM mode is only possible with two conductors and cannot exist in a waveguide. So, waveguides of a closed cross section with only one conductor can not support TEM waves.

- We conclude that a hollow waveguide consisting of a single conductor can not support TEM waves. If we have a hollow waveguide with a center conductor such as in a coaxial cable, however, TEM waves are supported. Other two-conductor waveguides such as striplines and two-wire lines also support these waves.
Rectangular WG: Waveguide modes

- The order of the mode refers to the field configuration in the guide, and is given by $m$ and $n$ integer subscripts, $TE_{mn}$ and $TM_{mn}$.
  - The $m$ subscript corresponds to the number of half-wave variations of the field in the $x$ direction, and
  - The $n$ subscript is the number of half-wave variations in the $y$ direction.
- A particular mode is only supported above its cutoff frequency. The cutoff frequency is given by

$$f_{cmn} = \frac{1}{2\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{c}{2\sqrt{\mu_r\varepsilon_r}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$u = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \frac{1}{\sqrt{\mu_r\varepsilon_r}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

----------(1)
Rectangular WG: Waveguide modes

Rectangular waveguide TE modes
Rectangular WG: TE Wave Considerations:

- Cutoff frequency can be obtained when $\gamma = 0$, i.e.

$$ (f_c)_{mn} = \frac{1}{2\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \quad \text{(Hz)} \quad \text{(1)} $$

- Cutoff wavelength will be

$$ (\lambda_c)_{mn} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}} \quad \text{(m)} \quad \text{(2)} $$

- For TE modes, in rectangular waveguides, either $m$ or $n$ can be zero, but not both.

- If $a > b$, the lowest possible mode is TE$_{10}$ (also the dominant mode), and the cutoff frequency is

$$ (f_c)_{TE_{10}} = \frac{1}{2a\sqrt{\mu\varepsilon}} = \frac{u}{2a} \quad \text{(1.1)} $$

TE$_{10}$ mode is the dominant mode as it has the lowest attenuation in a rectangular waveguide.
Rectangular WG: TE Wave Considerations

For a TE\(_{mn}\) mode the cut-off frequency is the frequency for which \(k_z = 0\). This means that the mode is in between its propagating and non-propagating stages. The cut off frequency for the TE\(_{mn}\) mode is

\[
\begin{equation}
 f_{c_{mn}} = \frac{c}{2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}.
\end{equation}
\]

The fundamental mode TE\(_{10}\)

The fundamental mode of a waveguide is the mode that has the lowest cut-off frequency. For a rectangular waveguide it is the TE\(_{10}\) mode that is the fundamental mode. It has \(f_{c10} = \frac{c}{2a}\). The electric field of the fundamental mode is

\[
E = E_0 \sin \left(\frac{\pi x}{a}\right) e^{-jk_z z} e_y.
\]

It is almost always the fundamental mode that is used in the waveguide. It is then crucial to make sure that the frequency is low enough such that only the fundamental mode can propagate. Otherwise there will be more than one mode in the waveguide and since the modes travel with different speeds, as will be seen below, one cannot control the phase of the wave.

Example: Consider \(a = 0.3\) m and \(b = 0.15\) m. Then \(f_{c10} = 500\) MHz, \(f_{c01} = f_{c20} = 1000\) MHz and \(f_{c11} = 1118\) MHz. It means that for frequencies lower than 500 MHz there are no waves that can propagate through the waveguide. In the interval 500 MHz < \(f\) < 1 GHz only the TE\(_{10}\) mode can propagate. One has to be in this frequency span in order to transfer a well defined signal.
Rectangular WG: TM Wave Considerations

- Cutoff frequency can be obtained when $\gamma = 0$, i.e.

\[
(f_c)_{mn} = \frac{1}{2} \frac{1}{\sqrt{\mu \varepsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \quad \text{(Hz)}
\]

- Cutoff wavelength will be

\[
(\lambda_c)_{mn} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}} \quad \text{(m)}
\]

- Wave impedance will be

\[
Z_{TM} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \quad \text{(\Omega)}
\]

- For TM modes in rectangular waveguides, neither $m$ nor $n$ can be zero as $m = 0$ or $n = 0$ will make all the cross-sectional field components vanish.

- The lowest possible mode is $TM_{11}$ in rectangular waveguides.
## Rectangular WG: Comparative features

<table>
<thead>
<tr>
<th>Mode</th>
<th>$Z$</th>
<th>$\lambda_g$</th>
<th>$u_p$</th>
<th>$u_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEM</td>
<td>$\eta$</td>
<td>$\lambda$</td>
<td>$u$</td>
<td>$u$</td>
</tr>
<tr>
<td>TM</td>
<td>$\eta \sqrt{1 - \left( \frac{f_c}{f} \right)^2} (\Omega)$</td>
<td>$\frac{\lambda}{\sqrt{1 - (f_c/f)^2}}$</td>
<td>$\frac{\lambda_g}{\lambda} u$</td>
<td>$\frac{\lambda}{\lambda_g} u$</td>
</tr>
<tr>
<td>TE</td>
<td>$\frac{\eta}{\sqrt{1 - (f_c/f)^2}}$</td>
<td>$\frac{\lambda}{\sqrt{1 - (f_c/f)^2}}$</td>
<td>$\frac{\lambda_g}{\lambda} u$</td>
<td>$\frac{\lambda}{\lambda_g} u$</td>
</tr>
</tbody>
</table>
Rectangular WG: Thumb Rules

There are a number of rules of thumb and common points that may be used when dealing with waveguide modes.

- For rectangular waveguides, the TE_{10} mode of propagation is the lowest mode that is supported.
- For rectangular waveguides, the width, i.e. the widest internal dimension of the cross section, determines the lower cut-off frequency and is equal to 1/2 wavelength of the lower cut-off frequency.
- For rectangular waveguides, the TE_{01} mode occurs when the height equals 1/2 wavelength of the cut-off frequency.
- For rectangular waveguides, the TE_{20} mode occurs when the width equals one wavelength of the lower cut-off frequency.
Problem-1: A hollow rectangular waveguide is to be used to transmit signals at a carrier frequency of 6 GHz. Choose its dimensions so that the cutoff frequency of the dominant TE mode is lower than the carrier by 25% and that of the next mode is at least 25% higher than the carrier.

Solution:
For \( m = 1 \) and \( n = 0 \) (TE\(_{10}\) mode) and \( u_{p_0} = c \) (hollow guide), Eq. (1) reduces to
\[
f_{10} = \frac{c}{2a}.
\]
Denote the carrier frequency as \( f_0 = 6 \) GHz. Setting
\[
f_{10} = 0.75f_0 = 0.75 \times 6 \text{ GHz} = 4.5 \text{ GHz},
\]
we have
\[
a = \frac{c}{2f_{10}} = \frac{3 \times 10^8}{2 \times 4.5 \times 10^9} = 3.33 \text{ cm}.
\]
If \( b \) is chosen such that \( a > b > \frac{a}{2} \), the second mode will be TE\(_{01}\). For TE\(_{01}\),
\[
f_{01} = \frac{c}{2b}.
\]
Setting \( f_{01} = 1.25f_0 = 7.5 \) GHz, we get
\[
b = \frac{c}{2f_{01}} = \frac{3 \times 10^8}{2 \times 7.5 \times 10^9} = 2 \text{ cm}.
\]
**Problem 2.** A waveguide, with dimensions \( a = 1 \) cm and \( b = 0.7 \) cm, is to be used at 20 GHz. Determine the wave impedance for the dominant mode when (a) the guide is empty, and (b) the guide is filled with polyethylene (whose \( \varepsilon_r = 2.25 \))

**Solution:**

For the TE_{10} mode,

\[
\frac{f_{10}}{2a} = \frac{c}{2a\sqrt{\varepsilon_r}}.
\]

When empty,

\[
f_{10} = \frac{3 \times 10^8}{2 \times 10^{-2}} = 15 \text{ GHz}.
\]

When filled with polyethylene, \( f_{10} = 10 \) GHz.

According to Eq.

\[
Z_{TE} = \frac{\eta}{\sqrt{1 - \left(\frac{f_{10}}{f}\right)^2}} = \frac{\eta_0}{\sqrt{\varepsilon_r} \sqrt{1 - \left(\frac{f_{10}}{f}\right)^2}}.
\]

When empty,

\[
Z_{TE} = \frac{377}{\sqrt{1 - (15/20)^2}} = 570 \Omega.
\]

When filled,

\[
Z_{TE} = \frac{377}{\sqrt{2.25\sqrt{1 - (10/20)^2}}} = 290 \Omega.
\]
Problem-3. A waveguide filled with a material whose $\varepsilon_r = 2.25$ has dimensions $a = 2$ cm and $b = 1.4$ cm. If the guide is to transmit 10.5-GHz signals, what possible modes can be used for the transmission?

Solution:
Application of Eq. gives:

$$u_{p0} = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{2.25}} = 2 \times 10^8 \text{ m/s},$$

$$f_{10} = 5 \text{ GHz (TE only)}$$
$$f_{01} = 7.14 \text{ GHz (TE only)}$$
$$f_{11} = 8.72 \text{ GHz (TE or TM)}$$
$$f_{20} = 10 \text{ GHz (TE only)}$$
$$f_{21} = 12.28 \text{ GHz (TE or TM)}$$
$$f_{12} = 15.1 \text{ GHz (TE or TM)}.$$

Hence, any one of the first four modes can be used to transmit 10.5-GHz signals.
References

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